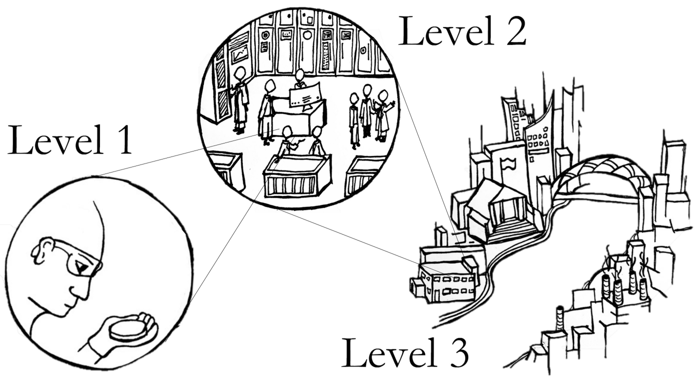
HPSC 1001/1901/2101/2901

WHAT IS THIS THING CALLED SCIENCE?

Semester 2, 2020

Lecture 19: Bayesianism and Modern Theories of Evidence

* Recall the different levels or degrees of 'zoom' from earlier: 1-2-3. Through the course, an increasing role for more social perspectives. Now we suddenly go back to level 1. And we find *perhaps* some resolution of earlier level 1 problems.



Imagine a dialogue with Kuhn, or Latour, about the role of evidence in scientific change. Part of the problem round 1970 was that outside of deductive logic, we don't even know what it would *look* like for evidence to be used rationally, whether or not people do actually behave that way. This has now changed.

* Presentation: backwards from T&R. Work through it both ways.

Two ideas in turn: (i) Belief. Is this a matter of degree?

I used that idea last week, in my handling of nonepistemic values in science. But does this make sense? What are these degrees? Feelings of confidence?

(ii) Probability and chance. Might evidence make a theory more *likely* to be true? But what is a probability? Hard to work out what they are, especially outside of situations where you have a repeated process of some kind (coin toss, etc.).

Against that background, Frank Ramsey's achievement:

1. Degrees of belief,
2. Interpreted as probabilities,
3. Updated via Bayes' theorem.

1. Degrees of belief

Belief and "acceptance" in C20 philosophy of science.
Some artificiality in the usual handling?
In science, certainty is just about impossible – everyone agrees with that. But "belief" is often handled in a yes or no way. This seems a mistake.

Many C20 views of evidence: you can *believe that* option X is the best supported, while believing that other options are also in play. OK - this is not a degree of belief view. It is an alternative. But let's look at the idea of degree of belief.

Degrees of belief are just feelings? Or a special kind of belief that complicates things?

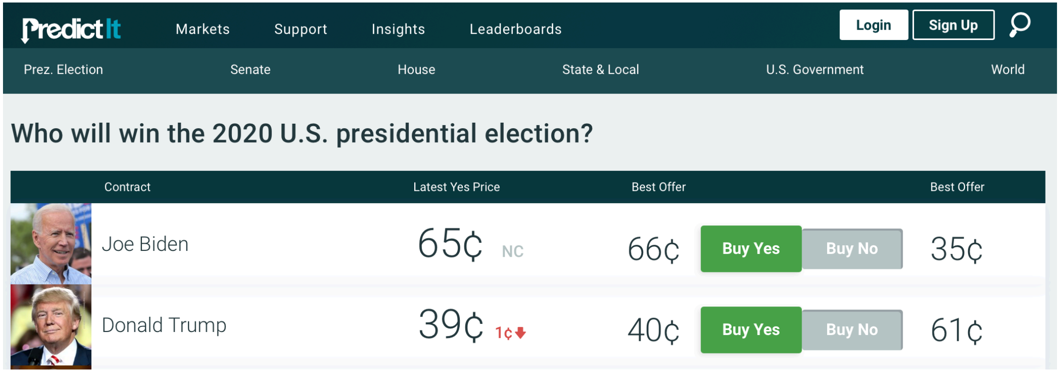
No. Frank Ramsey (1903-1930).
"Truth and Probability," 1926. Published after his death.
Degrees of belief are revealed in behavior.
More exactly: behavior is a consequence of degrees of belief along with preferences.
What behaviors reveal degrees of belief. Gambling is the easy case. But perhaps all life can be seen as a series of gambles. When you act, you bet that things will work out OK, always in the face of some degree of uncertainty.
Prediction markets - a good illustration.

PredictIt website. https://www.predictit.org/

You make predictions by buying shares. The price of a share, between 1 and 99 cents, corresponds to the market's estimate of the probability of an event taking place. [* Nb: probabilities can be 0 or 1 also.] Buy 'Yes' shares when the price is too low, when you think your fellow traders are underestimating this likelihood. Buy 'No' shares when you think they are too optimistic.

The value of your shares will change over time. You may decide to sell your shares later on, either to take some profit or stop a loss. Or, you can hold onto your shares until the market closes. At that point, if the event in the market has taken place, we'll redeem 'Yes' shares at $1. If it has not, 'No' shares will have that $1 value instead.

[me] How much you will pay is dependent on your degree of belief that the event will happen. Your degree of belief is manifested in behavior.



You might also bet on "Is there life on Venus?" I could not find a prediction market covering it. (PredictIt seems now to dominate, perhaps as the US has decided to allow it to operate, with constraints.) You could also bet on "My car is where I left it."

Another example, via Ramsey): How far would you walk to get directions, when you are not sure if you are on the right road? See end of these slides for details.

1. Degrees of belief,

2. Interpreted as probabilities,

What are probabilities? Chances of things happening? Chances of things being true?

There seems to be some sort of link to questions about evidence. Evidence can make something more likely...?

This was a big project in C20 empiricism (especially Carnap). It went badly.
Ramsey had, back in 1926, a novel idea.

Ramsey: interpret degrees of belief as probabilities.

"Subjective interpretation of probability." A probability is a measure of your *strength of confidence* in something, on a scale from 0 to 1 (inclusive).

Put degrees of belief on a scale from 0 to 1.
And: if *A* and *B* are exclusive (can't both be true) then your degree of belief in *A or B* is your degree of belief in *A* plus your degree of belief in *B*.
That is (nearly) all you need to interpret degrees of belief as probabilities.

P(*h*) = 1: you are completely certain that *h* is true.
P(*h*) = 1/2 : you are evenly balanced between believing that *h* is true and believing it is false.

** All this is just about *you*, not some property of *h* that exists independently of you. Talk about probability (on this view) is talk about levels of confidence.

Ramsey: your degrees of belief need to obey the rules of probability theory or you can be taken advantage of (in very special situations).

See *T&R* Ch. 14 on this -- the "Dutch book" proof.

2. Interpreted as probabilities, and...

3. Updated via Bayes' theorem

Evidence can rationally change your degrees of belief.

An old result with a new role. *Bayes' theorem.*
Thomas Bayes, C18.

First, introduce *conditional probability*. The probability of one thing *conditional on* another thing – or *assuming* another.
$P(X \mid Y)$: probability of X conditional on Y.

In a conditional probability $P(X|Y)$, we *ignore* whether you are certain or uncertain about Y. Just *assume* Y, and ask about the probability of X given Y.

What is the probability that a person is infected with Sars-Cov 2, *given* that their test is positive?
What is the probability that Biden wins, *given* that the election goes ahead without major disruptions?

$P(X|Y)$ is not the same as $P(Y|X)$.

Though the numbers might be the same in a particular case.

P(black card) = 1/2

P(black card | queen of clubs) = 1.

P(queen of clubs | black card) = 1/26 (no jokers)

Next: Bayes' theorem.
An 18th century result. Controversial for many years.
Always looked like it might help with evidence. But seemed to also lead to problems. It has a new role within "subjectivist" interpretations of probability.
Hence: "subjectivist Bayesianism."

An example. Suppose you think:

P(Pat is at the party) = 1/2

P(Pat's car is outside | Pat is at the party) = 0.8

P(Pat's car is outside | Pat is not at the party) = 0.01

P(Pat is at the party | Pat's car is outside) = ?

Bayes' theorem:

$$P(X|Y) = \frac{P(X) * P(Y|X)}{P(Y)}$$

Proof is very simple (once you have modern symbolism). See *T&R* Ch 14.

Applied to evidence:

How likely are we to see e, if h is true?

$$P(\text{hypothesis } h \mid \text{evidence } e) = \frac{P(h) * P(e|h)}{P(e)}$$

Without considering the new evidence, what is the overall or "prior" probability of h?

How likely are we to see e, overall? (Whether or not h is true). This will get explained further in a moment.

More on P(e): We want to take into account how likely is if h is true, and also how likely e is if h is false.

$$P(e) = Pe|h) * P(h) + P(e|\ \text{not}\ h) * P(\text{not}\ h)$$

What is P(h)? In older views of probability, there was much discussion of whether these "prior probabilities" of theories make sense at all. What do they measure? Maybe Bayes's theorem cannot be used in philosophy of science, because those prior probabilities of theories make no sense.

If you are a subjectivist, following Ramsey, this is no big deal. A "prior probability in h" is just your degree of belief in h before the evidence comes in. Fine.

Now we have all we need to solve some problems of evidence.

Hypothesis h is that Pat is at the party.
The evidence e is seeing Pat's car.

Do the maths from the case above:

$$P(h|e) = (0.8)(.5)/((0.8)(0.5) + (0.01)*(0.5)) = \text{(approx) } 0.99$$

When I did the example I did not think the number would come out quite so high. That 0.01 really has an effect; it would be hard for her car to be out there without her being inside.

The evidence *confirms* the hypothesis: $P(h|e) > P(h)$

$P(h|e)$ is called the *posterior* probability of h.
$P(h)$ is called the *prior* probability of h.
$P(e|h)$ and $P(e|\text{not } h)$ are both called *likelihoods*.

Suppose you thought she was very unlikely to show up:
P(Pat is at the party) = 0.1.
Then:
P(Pat is at the party | Pat's car is outside) =(approx) 0.89
Still very high! Again, that 0.01 really has an effect.

MORE: Ramsey's walking example, in detail.

You are walking somewhere, and meet a fork in road. You choose one fork and are about to start down it, but are not sure it's right. You see someone some distance away, off the road in the fields, who you take to be a local. You *could* then walk out of your way to ask that person if your road is right. Should you? It depends on how bad it would be if you were on the wrong road, and also on how much it costs to you (in time, energy, whatever) to walk over to ask the local.

And it also depends on *how confident you are* that you are already on the correct road.
Ramsey says: "I propose therefore to use the distance I would be prepared to go to ask, as a measure of the confidence of my opinion" that I have chosen the right road.
That is: the maximum distance you'd go to ask directions (walking to the person in the fields and then back), given the other costs and benefits etc., is a measure of your confidence.

If you are pretty sure you are right already, you will gain less from the walk if you do it (it will probably not change your mind, you think), so you'd not walk as far as you would if you were very uncertain. If you have no idea which fork is right, you'll be prepared to walk further to get information.

Let's do the maths. Assume C is your degree of belief that you are right already, before asking. Assume d is the max distance you'll walk to ask and get perfect information, $f(x)$ is the cost of walking x yards, r is the benefit of getting to the right place and w is the (lesser) benefit of getting to the wrong place. The maximum distance you will walk to get directions is a measure of how sure you are that you are right.

$$C = 1 - \frac{f(d)}{(r - w)}$$

(We have to assume that the value of $(f(d)/(r - w))$ is between 0 and 1.

Example: assume $r - w = 100$ units of value. The max distance you will walk is 100 yards. For each yard walked, you pay 1/10 unit of value. So $f(x) = x/10$.

If you walk 100 yards, the walking costs you 10 units of value.

Then we have:

$$C = 1 - \frac{10}{100}$$

$$= 0.9$$

So if 100 yards as the furthest you would walk, in this situation, your degree of belief that you are on the right road before you start must be 0.9.

(In this scenario, values of C less than 0.5 don't really make sense. You would be on the other road already, if your degree of belief that you had made the right choice was less than

0.5. That's OK. It's a simple case used to introduce the main ideas. Also: what if you do/don't like talking to strangers? These costs or benefits could be factored in.)